5b)

$$A = \int_0^{\frac{1}{2}} \left(xe^{-2x} - \frac{1}{2e}\right) dx$$

$$= \int_0^{\frac{1}{2}} xe^{-2x} - \int_0^{\frac{1}{2}} \frac{1}{2e} dx$$

$$\Rightarrow \int_0^{\frac{1}{2}} xe^{-2x} \qquad u = -2x, \quad dx \quad \frac{-du}{2}$$

from 0 to $-2(1/2)$

$$= -1\int_0^{-1} \frac{u}{2} e^{u} \frac{du}{2} \qquad = \frac{-1}{4}\int_0^{1} ue^{u}\, du$$

Apply integration by parts:

$$= -\frac{1}{4}\left[e^{u}u - \int e^{u} du\right]_1^0 \qquad = \frac{1}{4}\left[ue^{u} - e^{u}\right]_1^0$$

$$= \frac{1}{4}\left(-1 + \frac{2}{e}\right)$$

$$\text{Take } \frac{1}{2e}\int_0^{\frac{1}{2}} dx = \left[\frac{x}{2e}\right]_0^{1/2} = \frac{x}{4e}$$

$$= \frac{1}{4}\left(-1 + \frac{2}{e}\right) + \frac{1}{4e}$$

$$= \frac{-1}{4} + \frac{2}{4e} + \frac{1}{4e}$$

$$= \frac{-e+3}{4e}$$

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Q8

$x = 6\sin t \quad ; \quad \frac{dx}{dt} = 6\cos t$

$y = 5\sin 2t = 10 \sin t \cos t$

Bounds $[0, \frac{\pi}{2}]$

$$R = \int_0^{\pi/2} 10\sin t \cos t \cdot 6\cos t$$

$$= \int_0^{\pi/2} 60 \cos^2 t \sin t \, dt$$

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